Pauta Certamen 1.

1] $x(t) = 0.35 \sin(\omega t + \delta)$ [m]

a) $x(0) = -0.080\,m \Rightarrow 0.35 \sin\delta = -0.08$

5 $v(0) = 2.1\,m/s \Rightarrow \sin\delta = -0.228$

$E = 6\,J$

$\delta = \boxed{-}0.23$ rad
3 1

b) $x(t) = 0.35 \sin(\omega t - 0.23)$

5 $v(t) = 0.35\,\omega \cos(\omega t - 0.23)$

$v(0) = 0.35\,\omega \cos(-0.23) = 2.1$

$$\omega = \frac{2.1}{0.35 \cos(-0.23)} = 6.16\ \frac{rad}{s}$$

Dado que $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = 1.02$ [s].
1

c) $a(t) = -0.35\,\omega^2 \sin(6.16\,t - 0.23)$

5 $a(t) = -\,13.28 \sin(6.16\,t \underbrace{-\,0.23}_{-1})$

$a(1) = +4.59\ m/s^2$.
1

d) $\omega^2 = \frac{k}{m} \Rightarrow k = \omega^2 m = (6.16)^2 \times m \ \ldots$

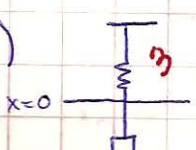
5

$$E = \frac{1}{2} k X_m^2 \rightarrow k = \frac{2E}{X_m^2} = \frac{2\times 6}{0.35^2} = 97.96 \left[\frac{N}{m}\right]$$
1

e) $m = \frac{k}{\omega^2} = \frac{97.96}{(6.16)^2} = 2.58$ kg.

5 1

f)

10 

$-kx + mg = ma$

$$-k\left[x + \frac{mg}{k}\right] = m\frac{d^2x}{dt^2}$$

$$X = x + \frac{mg}{k} \rightarrow \frac{d^2X}{dt^2} = \frac{d^2x}{dt^2}$$

Posición equilibrio

$a = 0 \Rightarrow x_i = -\frac{mg}{k}$

el resorte se estiraría

$x_i = 2.58 \times 10^{-1}$ [cm]

3]

$$f(t,z) = 0.5\left[\frac{V}{m}\right] \cos(kz + \omega t - \phi)$$

$$f(t,z) = 0.5\left[\frac{V}{m}\right] \cos\left(\frac{2\pi}{\lambda} z + \frac{2\pi}{T} t - \phi\right)$$

$$f(t,z) = 0.5 \frac{V}{m} \cos\left(\frac{2\pi}{\lambda}(z + vt) - \phi\right)$$

$$f(t,z) = 0.5 \frac{V}{m} \cos\left[\frac{2\pi}{670\times10^{-9}}(z + 3\times10^{8} t) - \phi\right]$$

para t en s y z en m.

$$f(0,\lambda/4) = 0.5 \frac{V}{m} \cos\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) - \phi\right]$$

$$\frac{\pi}{2} - \phi = 0 \Rightarrow \phi = \frac{\pi}{2}.$$

$$f(t,z) = 0.5 \frac{V}{m} \cos\left[\frac{2\pi}{670\times10^{-9}}(z + 3\times10^{8} t) - \frac{\pi}{2}\right]$$

5 5 5 5.

La diferencia con $g(t,z)$ es:

5 → la longitud de onda.
5 → la dirección de viaje de la onda
5 → la constante de fase, ambas ondas están desfasadas $\pi/2$ rad.

2] $x(t) = x_m e^{-\eta t/2} \cos(\omega' t - \phi)$ $\boxed{t[s]}$.

$x_m = \underset{cm}{1-10}$; $\eta = \underset{s}{0-1}$ $\phi = \pi, \pi/2, 0$; $\omega' = 0, \pi, 2\pi, 3\pi$.

$\omega' = \sqrt{\frac{k}{m} - \frac{\eta^2}{4}}$

$x(1.5) = 6\ cm \rightarrow$ máx. n

$x(2.5) = 5.6\ cm \rightarrow$ máx $n+1$

Si el valor es máximo $\Rightarrow |\cos(\omega' t - \phi)| = 1$.

$$\begin{cases} \omega' t_1 - \phi = 2n\pi \\ \omega' t_2 - \phi = 2(n+1)\pi \end{cases} \begin{pmatrix} \rightarrow 0 \\ \rightarrow 2\pi \end{pmatrix}$$

$(-)$ $\omega'(t_1 - t_2) = 2\cancel{n}\pi - 2(n+1)\pi$

$$\omega'(t_1 - t_2) = -2\pi$$

$$\omega'(1.5 - 2.5) = -2\pi \Rightarrow \boxed{\omega' = 2\pi} \left[\frac{rad}{s}\right] \quad 5$$

$\Rightarrow$ $2\pi \frac{3}{2} - \phi = 2n\pi$.

$$\phi = 2\pi \times 1.5 - 2n\pi$$

$$\phi = \pi(3 - 2n) = 3\pi, \pi, -\pi, \ldots$$

$$\boxed{\phi = \pi \pm 2n\pi} \quad 5$$

$x(1.5) = x_m e^{-\eta \times 1.5/2} = 6\ cm$.

$x(2.5) = x_m e^{-\eta \times 2.5/2} = 5.6\ cm$

$(\%)$ $e^{-\eta/2(1.5-2.5)} = \frac{6}{5.6} \Rightarrow e^{+0.5\eta} = 1.07$.

$0.5\eta = \ln 1.07 = 6.9 \times 10^{-2}$

$\boxed{\eta = 1.38 \times 10^{-1}\ s^{-1}}$ 5

$x_m e^{-\frac{1.38 \times 10^{-1}}{2} \times 1.5} = 6$

$\Rightarrow x_m = 6.65\ cm$ 5 $\Rightarrow x(0) = -6.65\ cm$. 5

$x(3) = -5.4\ cm$ 5