DEPARTMENT OF MATHEMATICS

Student's Name__________ Course#______ Prob.#______ Date______

Section Instructor__________ Sec.#______

GRADING

HONOR PLEDGE: I pledge on my honor that I have not given or received any unauthorized assistance on this examination or assignment.

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P1

(1) $\langle \cdot,\cdot \rangle$ is bilinear and symmetric ✓

Let $p(x) = a + bx + cx^2 \in \mathbb{P}^2$, then

$\langle p,p \rangle = p(-1)^2 + p(0)^2 + p(1)^2 \geq 0$ and

$\langle p,p \rangle = 0 \Leftrightarrow p(-1) = p(0) = p(1) = 0$

$$\Rightarrow \left.\begin{array}{l} a - b + c = 0 \\ a = 0 \\ a + b + c = 0 \end{array}\right\} \Rightarrow p = \theta$$

(2) $p(x) = 1$, $q(x) = x$: $\langle p,q \rangle = -1 + 0 + 1 = 0$

(3) $\langle r, p \rangle = 0 \Rightarrow r(-1) + r(0) + r(1) = 0$

$\Rightarrow 3c + 2 = 0$

$\langle r, q \rangle = 0 \Rightarrow \langle x^2, x \rangle + b\langle x,x \rangle + c\langle x,1 \rangle = 0$

$2b = 0$

$\Rightarrow r(x) = x^2 - \frac{2}{3}$

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Pb 2/

$$q(x_i) = y_i \Rightarrow \frac{1}{q(x_i)} = \frac{1}{y_i}$$

$$\Rightarrow a(3+x_i) + b\sin(x-\pi) = \frac{1}{y_i}$$

$$\begin{bmatrix} 3+1 & \sin(1-\pi) \\ 3+2 & \sin(2-\pi) \\ 3+3 & \sin(3-\pi) \\ 3+4 & \sin(4-\pi) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{1}{10} \\ \frac{1}{15} \\ \frac{1}{17} \\ \frac{1}{23} \end{bmatrix}$$

$$A \qquad x \qquad \bar{b}$$

Normal eq. $\quad A^T A x = A^T b.$

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Pb 3/

(1) $p \in \text{span}\{t, t^2\}$ $\quad f(t) = e^t \cos(t)$

$$\begin{cases} \langle p - f, w \rangle = 0 & \forall w \in \text{Span}\{t, t^2\} \\ p = \alpha_1 t + \alpha_2 t^2 \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} \langle t, t \rangle & \langle t^2, t \rangle \\ \langle t, t^2 \rangle & \langle t^2, t^2 \rangle \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \langle e^t \cos(t), t \rangle \\ \langle e^t \cos(t), t^2 \rangle \end{bmatrix}}_{b}$$

(2) Matrix A is the Gram matrix of a lin indep. set $\{t, t^2\}$ $\Rightarrow$ A is SPD

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Pb 4 (1) $A = BB^T$ is symmetric, moreover

$$x^T A x = x^T B B^T x = (B^T x)^T B^T x = \|B^T x\|^2 \geq 0 \quad \forall x \in \mathbb{R}^n$$

if $x^T A x = 0 \Rightarrow \|B^T x\|^2 = 0$

$\Rightarrow x = \theta$ iff B^T is nonsingular

(2)

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 8 & 4 \\ 0 & 4 & 8 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 8 \end{pmatrix} \xrightarrow{R_3 - R_2} \underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 0 & 4 & 4 \\ 0 & 0 & 4 \end{pmatrix}}_{U}$$

A symmetric and the pivots of U are $> 0 \Rightarrow$ A is SPD.

Decomp $A = LU$ $\quad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ $\quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

$= LDL^T$

$= (LD^{\frac{1}{2}})(LD^{\frac{1}{2}})^T$ $\quad D^{\frac{1}{2}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$= G\,G^T$

↳ Cholesky decomp.

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Pb 5/ (1) $S = \text{span}\{a_1, a_2, a_3\}$ $a_1 = (0,-1,1,0)^T$

$a_2 = (2,0,2,0)^T$ $a_3 = (-1,0,0,1)^T$

Gram-Schmidt:

$v_1 = a_1$

$v_2 = a_2 - \frac{\langle a_2, v_1\rangle}{\|v_1\|^2} v_1$

$v_3 = a_3 - \frac{\langle a_3, v_1\rangle}{\|v_1\|^2} v_1 - \frac{\langle a_3, v_2\rangle}{\|v_2\|^2} v_2$

$v_1 = a_1 = (0,-1,1,0)^T$

$v_2 = a_2 - 1 v_1 = (2\ 1\ 1\ 0)^T$

$v_3 = a_3 - 0\ v_1 + \frac{1}{3} v_2 = (-\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 1)^T$

$\Rightarrow S = \text{span}\{v_1, v_2, v_3\}$

(2) Proj of $b = (-1,1,1,1)^T$ onto $S = \text{span}\{v_1, v_2, v_3\}$

$$\text{Proj } b = w = \frac{\langle b, v_1\rangle}{\|v_1\|^2} v_1 + \frac{\langle b, v_2\rangle}{\|v_2\|^2} v_2 + \frac{\langle b, v_3\rangle}{\|v_3\|^2} v_3$$

$$= 0 \cdot v_1 + 0 \cdot v_2 + \frac{2}{\frac{4}{3}} v_3$$

$$= \frac{3}{2} v_3$$