Math 401: Sec 0501: Homework 5

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Due: Mar. 27, 2015

Complete problems 1–5. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 2.2.22 : Show that if W and Z are subspaces of V, then

a) their intersection $W \cap Z$ is a subspace of V,

b) their sum $W + Z = \{w + z : w \in W, z \in Z\}$ is also a subspace, but

c) their union $W \cup Z$ is not a subspace of V, unless $W \subset Z$ or $Z \subset W$.

Problem 2: Let V be a vector space and $S \subset V$. Determine if S is a basis of V when:

a) $S := \{(0, 4, -1), (-1, 0, 1), (1, -8, 1)\}$ and $V := \mathbb{R}^3$.

b) $S := \{x^2 + 1, x^2 - 1, x^2 + x + 1\}$ and $V := \mathbb{P}^2$.

c) $S := \{x^3, x^2 + 1, x^2 - x, x + 1\}$ and $V := \mathbb{P}^3$.

Problem 2.3.7: Let S be the subspace of $\mathbb{R}^{2\times 2}$ consisting of all symmetric matrices. Show that S is spanned by the matrices $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Problem 2.4.15: Determine all values of the scalar k for which the following four matrices form a basis for $\mathbb{R}^{2\times 2}$:

$$A_1 = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \qquad A_2 = \begin{pmatrix} k & -3 \\ 1 & 0 \end{pmatrix} \qquad A_3 = \begin{pmatrix} 1 & 0 \\ -k & 2 \end{pmatrix} \qquad A_4 = \begin{pmatrix} 0 & k \\ -1 & -2 \end{pmatrix}.$$

Problem 2.5.24: For each of the following matrices A:

a) Determine the rank and the dimensions of the range and kernel.

b) Find basis for both the range and kernel.

c) Find explicit conditions on vector $\boldsymbol{b}$ which guarantee that the system $A\boldsymbol{x} = \boldsymbol{b}$ has a solution.

d) Write down a specific *nonzero* vector $\boldsymbol{b}$ that satisfies your conditions, and then find all possible solutions $\boldsymbol{x}$.

$$i) \begin{pmatrix} 2 & -5 & -1 \\ 1 & -6 & -4 \\ 3 & -4 & 2 \end{pmatrix} \qquad\qquad ii) \begin{pmatrix} 2 & 5 & 7 \\ 6 & 13 & 19 \\ 3 & 8 & 11 \\ 1 & 2 & 3 \end{pmatrix}$$