

Math 401: Sec 0501: Homework 4

Pablo Venegas

Handed out: Feb. 20, 2015

Due: Feb. 27, 2015

Complete problems 1–3. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 1.8.4 (40 pts.): Let

$$A = \left(\begin{array}{ccc|c} a & 0 & b & 2 \\ a & 2 & a & b \\ b & 2 & a & a \end{array} \right)$$

be the augmented matrix for a linear system. For which values of a and b does the system have

- i) a unique solution?.
- ii) no solution?.
- iii) infinitely many solutions?.
- iv) Given $a = -2$ and $b = 0$, find all solutions of the system.

Problem 2.1 (20 pts.): Determine which of the following sets are vector spaces. If the set is not a vector space explain which property is not satisfied. On the other hand, if the set is a vector space you should provide a proof.

- a) $V = \{A \in \mathbb{R}^{n \times n} : A \text{ is a diagonal matrix}\}.$
- b) $V = \left\{ A \in \mathbb{R}^{2 \times 2} : A = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}, \quad a, b \in \mathbb{R} \right\}.$
- c) $V = \{(x, y, z) \in \mathbb{R}^3 : x \geq 0, y \geq 0, z \geq 0\}.$
- d) $V = \{(x, y, z) \in \mathbb{R}^3 : yz = 0\}.$

Problem 2.2.13 (40 pts.): Which of the following are subspaces? Justify your answer!

- a) The set of all row vectors of the form $(a, 3a).$
- b) The set of all vectors of the form $(a, a + 1).$
- c) The set of all continuous functions in \mathbb{R} for which $f(-1) = 0.$
- d) The set of all periodic functions of period 1, i.e. $f(x + 1) = f(x).$
- e) The set of all non-negative functions, i.e. $f(x) \geq 0.$
- f) The set of all even polynomials, i.e. $p(x) = p(-x).$
- g) The set of all polynomials $p(x)$ that have $(x - 1)$ as a factor.
- h) The set of all quadratic forms $q(x, y) = ax^2 + bxy + cy^2.$