Math 401: Sec 0501: Homework 3

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Due: Feb. 20, 2015

Complete problems 1–5. Each of these problems is worth 20 points. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 1.4: Let $A \in \mathbb{R}^{3\times 3}$ be

$$A = \begin{pmatrix} 1 & a & 1 \\ a & 1 & 1 \\ b & ab+1 & 0 \end{pmatrix} \qquad a, b \in \mathbb{R}$$

a) For which real numbers a and b matrix A is regular?.

b) Given $a = -1$ and $b = 1$, perform the permuted LU decomposition of A.

Problem 1.5.17: Prove that if $U \in \mathbb{R}^{n\times n}$ is a nonsingular upper triangular matrix, then the diagonal entries of U^{-1} are the reciprocals of the diagonal entries of U.

Problem 1.5.24.h: Find the inverse of the following matrix, if possible, by applying the Gauss-Jordan Method.

$$A = \begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & -2 & -5 \end{pmatrix}$$

Problem 1.6.13: Let $A, B \in \mathbb{R}^{m\times n}$.

(a) Suppose that $\mathbf{v}^T A\mathbf{w} = \mathbf{v}^T B\mathbf{w}$ for all vectors $\mathbf{v} \in \mathbb{R}^m$ and $\mathbf{w} \in \mathbb{R}^n$. Prove that $A = B$.

(b) Give an example of two matrices A and B such that $\mathbf{v}^T A\mathbf{v} = \mathbf{v}^T B\mathbf{v}$ for all vectors $\mathbf{v}$, but $A \neq B$.

Problem 1.7: Let $U = \{u_{ij}\}\ i, j = 1, \ldots, n$ be a nonsingular upper triangular matrix. The solution of the system $U\mathbf{x} = \mathbf{b}$ is given by the *Back Substitution* algorithm:

For $i = n, n-1, \ldots, 1$

$$x_i = \frac{1}{u_{ii}} \left(b_i - \sum_{j=i+1}^{n} u_{ij} x_j \right)$$

(a) Perform an operation count for the algorithm above.

(b) Let A be a general $n \times n$ regular matrix. Determine the exact number of operations needed to compute the solution of $A\mathbf{x} = \mathbf{b}$ using the LU decomposition of A.