Math 401: Sec 0501: Homework 1

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Handed out: Jan. 30, 2015
Due: Feb. 6, 2015

Complete problems 1–5. Each of these problems is worth 20 points. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 1.1.1.d: Solve the following system of linear equations by reducing to triangular form and then using *Back Substitution*.

$$\begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & 3 \\ 3 & 0 & -2 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

Problem 1.1.2: How should the coefficients a, b and c be chosen so that the system $ax + by + cz = 3$, $ax - y + cz = 1$, $x + by - cz = 2$, has the solution $x = 1$, $y = 2$ and $z = -1$?.

Problem 1.2.12:

(a) Show that if $D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ is a 2×2 diagonal matrix with $a \neq b$, then the only matrices that commute (under matrix multiplication) with D are another 2×2 diagonal matrices.

(b) What if $a = b$?

(c) Find all the matrices that commute with $D = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ where a, b, c are different.

(d) Answer the same question for the case when $a \neq b = c$.

Problem 1.2.15:.

(a) Show that, if A and B are commuting square matrices, then $(A + B)^2 = A^2 + 2AB + B^2$.

(b) Find a pair of 2×2 matrices A and B such that $(A + B)^2 \neq A^2 + 2AB + B^2$.

Problem 1.2.32: The *trace* of a $n \times n$ matrix $A \in \mathbb{R}^{n \times n}$ is defined to be the sum of its diagonal entries: $\operatorname{tr}(A) = a_{11} + a_{22} + a_{33} + \cdots + a_{nn}$.

(a) Compute the trace of

$$\text{(i)} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \qquad \text{(ii)} \begin{pmatrix} 1 & 3 & 2 \\ -1 & 0 & 1 \\ -4 & -3 & -1 \end{pmatrix}$$

(b) Prove that $\operatorname{tr}(A + B)$= $\operatorname{tr}(A)$ + $\operatorname{tr}(B)$.

(c) Prove that $\operatorname{tr}(AB)$= $\operatorname{tr}(BA)$.

(d) Prove that the commutator matrix $C = AB - BA$ has zero trace $\operatorname{tr}(C)$= 0.

(e) Is part (c) valid if A has size $m \times n$ and B has size $n \times m$?.